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THE SCATTERING OF HARMONIC ELASTIC ANTI-PLANE SHEAR WAVES BY TWO COLLINEAR CRACKS IN THE PIEZOELECTRIC PLATE BY USING THE NON-LOCAL THEORY*

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ABSTRACT: In this paper, the dynamic interaction between two collinear cracks in a piezoelectric material plate under anti-plane shear waves is investigated by using the non-local theory for impermeable crack surface conditions. By using the Fourier transform, the problem can be solved with the help of two pairs of triple integral equations. These equations are solved using the Schmidt method. This method is more reasonable and more appropriate. Unlike the classical elasticity solution, it is found that no stress and electric displacement singularity is present at the crack tip. The non-local dynamic elastic solutions yield a finite hoop stress at the crack tip, thus allowing for a fracture criterion based on the maximum dynamic stress hypothesis.

KEY WORDS: elastic waves, crack, piezoelectric materials, non-local theory

1 INTRODUCTION

In the theoretical studies of crack problems for the piezoelectric materials, several different electric boundary conditions at the crack surfaces have been proposed by numerous researchers[1~11]. For the sake of analytical simplification, the assumption that the crack surfaces are impermeable to electric fields was adopted in Refs.[1~5, 12,13]. In their models, the assumption of the impermeable cracks refers to the fact that the crack surfaces are free of surface charge and thus the electric displacement vanishes insides the crack. However, these solutions contain stress and electric displacement singularities. This is not reasonable according to the physical nature. To overcome the stress singularity in the classical elastic theory, the non-local theory was used to study the state of stress near the tip of a sharp line crack in an elastic plate subject to uniform tension, shear and anti-plane shear in Refs.[14~16]. The obtained solutions did not contain any stress singularity.

In the present paper, the scattering of harmonic elastic anti-plane shear waves by two collinear symmetrical impermeable cracks in piezoelectric materials is investigated by using the non-local theory. This problem is different from the one in Ref.[17], which the static fracture problem was investigated for a permeable crack in the piezoelectric materials by using the non-local theory. To overcome the mathematical difficulties, one has to accept some assumptions as in Refs.[18,19], where one-dimensional non-local kernel function is used instead of two-dimensional kernel function for the anti-plane dynamic problem to obtain the stress and electric displacement at the crack tips. These assumptions have been used in the previous studies[20,21]. Certainly, the assumption should be further investigated against the realistic conditions. The Fourier transform is applied and a mixed boundary value problem is reduced to two pairs of triple integral equations. In solving the triple integral equations, the crack surface displacement and electric potential are expanded in a series of Jacobi polynomials.

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This process is quite different from that adopted in Refs.[1~16]. As expected, the solution in this paper does not contain the stress and electric displacement singularities at the crack tip, thus clearly reflecting the physical nature of the problem.

2 BASIC EQUATIONS OF NON-LOCAL PIEZOELECTRIC MATERIALS

For the anti-plane shear problem, the basic equations of linear, non-local piezoelectric materials, with vanishing body force are[13,14]

$$\begin{aligned}&\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}=\rho\frac{\partial^2 w}{\partial t^2}\\&\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}=0\end{aligned} \tag{1}$$

$$\begin{aligned}\tau_{kz}(X,t)=\int_V[&c'_{44}(|X'-X|)w_{'k}(X',t)+\\&e'_{15}(|X'-X|)\phi_{'k}(X',t)]\mathrm{d}V(X') \qquad k=x,y\end{aligned} \tag{2}$$

$$\begin{aligned}D_k(X,t)=\int_V[&e'_{15}(|X'-X|)w_{'k}(X',t)-\\&\varepsilon'_{11}(|X'-X|)\phi_{'k}(X',t)]\mathrm{d}V(X') \qquad k=x,y\end{aligned} \tag{3}$$

where the only difference as compared with the classical elastic theory and the piezoelectric theory is in the stress and the electric displacement constitutive Eqs.(2), (3) in which the stress $\tau_{zk}(X,t)$ and the electric displacement $D_k(X,t)$ at a point X depends on $w_{'k}(X,t)$ and $\phi_{'k}(X,t)$, at all points of the body. w and ϕ are the mechanical displacement and electric potential. For homogeneous and isotropic piezoelectric materials there exist only three material parameters, $c'_{44}(|X'-X|)$, $e'_{15}(|X'-X|)$ and $\varepsilon'_{11}(|X'-X|)$ which are functions of the distance $|X'-X|$. ρ is the density of the piezoelectric materials. The integrals in Eqs.(2), (3) are over the volume V of the body enclosed within a surface ∂V. As discussed in the papers[20,21], the form of $c'_{44}(|X'-X|)$, $e'_{15}(|X'-X|)$ and $\varepsilon'_{11}(|X'-X|)$ for which the dispersion curves of plane elastic waves coincide with those known in lattice dynamics. Among several possible curves the following has been found to be very useful

$$(c'_{44},e'_{15},\varepsilon'_{11})=(c_{44},e_{15},\varepsilon_{11})\alpha(|X'-X|) \tag{4}$$

$\alpha(|X'-X|)$ is known as influence function, and is the functions of the distance $|X'-X|$. $c_{44}, e_{15}, \varepsilon_{11}$ are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively.

Substitution of Eq.(4) into Eqs.(2), (3) yields

$$\tau_{kz}(X,t)=\int_V\alpha(|X'-X|)\sigma_{kz}(X',t)\mathrm{d}V(X') \qquad k=x,y \tag{5}$$

$$D_k(X,t)=\int_V\alpha(|X'-X|)D^c_k(X',t)\mathrm{d}V(X') \qquad k=x,y \tag{6}$$

where

$$\sigma_{kz}=c_{44}w_{'k}+e_{15}\phi_{'k} \qquad D^c_k=e_{15}w_{'k}-\varepsilon_{11}\phi_{'k} \tag{7}$$

The expressions (7) are the classical constitutive equations.

3 THE CRACK MODEL

It is assumed that there are two collinear symmetric cracks of length $1-b$ along the x-axis in the piezoelectric material plate as shown in Fig.1. $2b$ is the distance between the two cracks.

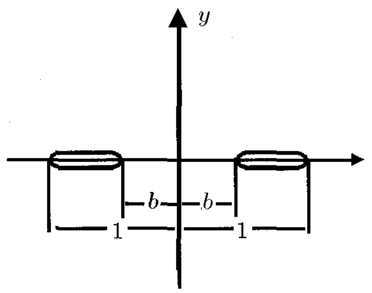

Fig.1 Cracks in the piezoelectric materials

In this paper, the harmonic anti-plane shear wave is vertically incident. Let ω be the circular frequency of the incident wave. $-\tau_0$ is a magnitude of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $\mathrm{e}^{-i\omega t}$ will be suppressed but understood. The solution of two collinear symmetric cracks of arbitrary finite length can easily be obtained by a simple change in the numerical values of the present problem. The piezoelectric boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement and the in-plane electric fields. When the cracks are subjected to the harmonic elastic waves and a constant electric displacement $D_y=-D_0$, as discussed by in Refs.[16, 22, 23], the boundary conditions on the crack faces at $y=0$ are (b is a dimensionless variable.)

$$\tau_{yz}(x,0,t)=-\tau_0 \qquad D_y(x,0,t)=-D_0$$

$$b \le |x| \le 1 \tag{8}$$

$$w(x,0,t) = \phi(x,0,t) = 0$$
$$|x| < b \qquad |x| > 1 \tag{9}$$

$$w(x,y,t) = \phi(x,y,t) = 0$$
$$\text{for } (x^2+y^2)^{1/2} \to \infty \tag{10}$$

Substituting Eqs.(5) and (6) into Eq.(1), respectively, using Green-Gauss theorem, one obtains[16]

$$\iint_V \alpha(|x'-x|,|y'-y|)[c_{44}\nabla^2 w(x',y',t)+ e_{15}\nabla^2\phi(x',y',t)]\mathrm{d}x'\mathrm{d}y' - \left(\int_{-1}^{-b} + \int_b^1\right)\alpha(|x'-x|,0)[\sigma_{yz}(x,0^+,t) - \sigma_{yz}(x,0^-,t)]\mathrm{d}x' = \rho\frac{\partial^2 w}{\partial t^2} \tag{11}$$

$$\iint_V \alpha(|x'-x|,|y'-y|)[e_{15}\nabla^2 w(x',y',t)- \varepsilon_{11}\nabla^2\phi(x',y',t)]\mathrm{d}x'\mathrm{d}y' - \left(\int_{-1}^{-b} + \int_b^1\right)\alpha(|x'-x|,0)[D_y^c(x,0^+,t) - D_y^c(x,0^-,t)]\mathrm{d}x' = 0 \tag{12}$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two dimensional Laplace operator. Under the applied anti-plane shear load on the unopened surfaces of the crack, the displacement field and the electric displacement have the following symmetry relations

$$\begin{aligned} w(x,-y,t) &= -w(x,y,t) \\ \phi(x,-y,t) &= -\phi(x,y,t) \end{aligned} \tag{13}$$

Using Eq.(13), we find that

$$\begin{aligned} &[\sigma_{yz}(x,0^+,t) - \sigma_{yz}(x,0^-,t)] = 0 \\ &[D_y^c(x,0^+,t) - D_y^c(x,0^-,t)] = 0 \end{aligned} \tag{14}$$

Hence, the line integrals in Eqs.(11) and (12) vanish. By carrying out the Fourier transform of Eqs.(11) and (12) with respect to x', it can be shown that

$$\int_0^\infty \bar{\alpha}(|s|,|y'-y|)\Big\{c_{44}\Big[\frac{\mathrm{d}^2\bar{w}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y',t)\Big] + e_{15}\Big[\frac{\mathrm{d}^2\bar{\phi}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y',t)\Big]\Big\}\mathrm{d}y' = -\rho\omega^2\bar{w} \tag{15}$$

$$\int_0^\infty \bar{\alpha}(|s|,|y'-y|)\Big\{e_{15}\Big[\frac{\mathrm{d}^2\bar{w}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y',t)\Big] - \varepsilon_{11}\Big[\frac{\mathrm{d}^2\bar{\phi}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y',t)\Big]\Big\}\mathrm{d}y' = 0 \tag{16}$$

Here a superposed bar indicates the Fourier transform, e.g.

$$\bar{f}(s,y) = \int_0^\infty f(x,y)\mathrm{e}^{\mathrm{i}sx}\mathrm{d}x$$

What now remains to be done is to solve the integrodifferential Eqs.(15) and (16) for the function w and ϕ. It seems obvious that a rigorous solution of such a problem encounters serious if not unsurmountable mathematical difficulties, and one has to resort to an approximate procedure. In the given problem, according to the assumptions as in Refs.[18,19], the non-local interaction in y direction was ignored. Therefore

$$\bar{\alpha}(|s|,|y'-y|) = \bar{\alpha}_0(s)\delta(y'-y) \tag{17}$$

From Eqs.(15) and (16), it can be shown that

$$\bar{\alpha}_0(s)\Big\{c_{44}\Big[\frac{\mathrm{d}^2\bar{w}(s,y,t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y,t)\Big] + e_{15}\Big[\frac{\mathrm{d}^2\bar{\phi}(s,y,t)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y,t)\Big]\Big\} = -\rho\omega^2\bar{w} \tag{18}$$

$$e_{15}\Big[\frac{\mathrm{d}^2\bar{w}(s,y,t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y,t)\Big] - \varepsilon_{11}\Big[\frac{\mathrm{d}^2\bar{\phi}(s,y,t)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y,t)\Big] = 0 \tag{19}$$

Because of symmetry, it suffices to consider the problem in the first quadrant only. The solution of Eqs.(18) and (19) does not present difficulties, which can be written as follows, respectively ($y \ge 0$)

$$\begin{aligned} &w(x,y,t) = \frac{2}{\pi}\int_0^\infty A(s)\mathrm{e}^{-\gamma y}\cos(xs)\mathrm{d}s \\ &\phi(x,y,t) - \frac{e_{15}}{\varepsilon_{11}}w(x,y,t) = \frac{2}{\pi}\int_0^\infty B(s)\mathrm{e}^{-sy}\cos(xs)\mathrm{d}s \end{aligned} \tag{20}$$

where $\gamma^2 = s^2 - \omega^2/c^2\bar{\alpha}_0(s)$, $c^2 = \mu/\rho$, $\mu = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}$. $A(s)$ and $B(s)$ are to be determined from the boundary conditions. According to the boundary conditions (8) and (9), one obtains

$$\frac{2}{\pi}\int_0^\infty \bar{\alpha}_0(s)\gamma A(s)\cos(sx)\mathrm{d}s = \frac{1}{\mu}\Big(\tau_0 + \frac{e_{15}D_0}{\varepsilon_{11}}\Big)$$

$$b \le |x| \le 1 \tag{21}$$

$$\frac{2}{\pi}\int_0^\infty A(s)\cos(sx)\mathrm{d}s = 0$$

$$|x| < b \qquad |x| > 1 \tag{22}$$

and

$$\frac{2}{\pi}\int_0^\infty \bar{\alpha}_0(s) sB(s)\cos(sx)\mathrm{d}s = -\frac{D_0}{\varepsilon_{11}}$$

$$b \le |x| \le 1 \tag{23}$$

$$\frac{2}{\pi}\int_0^\infty B(s)\cos(sx)\mathrm{d}s = 0$$

$$|x| < b \qquad |x| > 1 \tag{24}$$

Equations (21)~(24) are the triple integral equations of this problem.

4 SOLUTION OF THE TRIPLE INTEGRAL EQUATIONS

The non-local function α will depend on a characteristic length ratio a/l, where a is an internal characteristic length (e.g., lattice parameter, granular distance. In this paper, a represents the lattice parameter.) and l is an external characteristic length (e.g., crack length, wave-length. In this paper, l represents the crack length $1-b$.). By matching the dispersion curves of plane waves with those of atomic lattice dynamics (or experiments), we can determine the non-local modulus function α for given material. Here, the only difference between the classical and non-local equations is in the introduction of the function $\bar{\alpha}_0(s)$. As discussed in Refs.[14~16, 18,19, 24], we have

$$\alpha_0 = \chi_0 \exp(-(\beta/a)^2(x'-x)^2) \quad \chi_0 = \beta/a\sqrt{\pi} \tag{25}$$

where β is a constant (here $\beta = e_0\sqrt{\pi}/(1-b)$, e_0 is a constant appropriate to each material.). a is the lattice parameter. So one obtains

$$\bar{\alpha}_0(s) = \exp(-(sa)^2/(2\beta)^2) \tag{26}$$

with $\bar{\alpha}_0(s) = 1$ for the limit $a \to 0$ (We consider the crystal as a lattice of regularly space sites with lattice parameter a), so that Eqs.(21)~(24) reduce to the well-known triple integral equations of the classical theory. The triple integral equations for the same problem in the classical fracture theory can be transformed into a Fredholm integral equation of the second kind. However, as discussed in Ref.[17], the triple integral Eqs.(21)~(24) cannot be transformed into a Fredholm integral equation of the second kind. This makes the numerical solution of such equations quite difficult. Here the Schmidt method[25] can be used to solve the triple integrals in Eqs.(21)~(24). The displacement w and the electric potential ϕ can be represented by the following series

$$w(x,0,t) = \sum_{n=0}^\infty a_n P_n^{(1/2,1/2)}\Big(\frac{x-(1+b)/2}{(1-b)/2}\Big)\cdot \Big(1-\frac{(x-(1+b)/2)^2}{((1-b)/2)^2}\Big)^{1/2}$$

$$\text{for } b \le x \le 1 \qquad y=0 \tag{27}$$

$$w(x,0,t) = 0$$

$$\text{for } x>1 \qquad x<b \qquad y=0 \tag{28}$$

$$\phi(x,0,t) = \sum_{n=0}^\infty b_n P_n^{(1/2,1/2)}\Big(\frac{x-(1+b)/2}{(1-b)/2}\Big)\cdot \Big(1-\frac{(x-(1+b)/2)^2}{((1-b)/2)^2}\Big)^{1/2}$$

$$\text{for } b \le x \le 1 \qquad y=0 \tag{29}$$

$$\phi(x,0,t) = 0$$

$$\text{for } x>1 \qquad x<b \qquad y=0 \tag{30}$$

where a_n and b_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial[26]. The Fourier transforms of Eqs.(27) and (29) are[27]

$$A(s) = \bar{w}(s,0,t) = \sum_{n=0}^\infty a_n B_n G_n(s)\frac{1}{s}\mathrm{J}_{n+1}\Big(s\frac{1-b}{2}\Big) \tag{31}$$

$$B(s) = \bar{\phi}(s,0,t) - \frac{e_{15}}{\varepsilon_{11}}\bar{w}(s,0,t) = \sum_{n=0}^\infty \Big(b_n - \frac{e_{15}}{\varepsilon_{11}}a_n\Big)\cdot B_n G_n(s)\frac{1}{s}\mathrm{J}_{n+1}\Big(s\frac{1-b}{2}\Big) \tag{32}$$

$$B_n = 2\sqrt{\pi}\frac{\Gamma\Big(n+1+\dfrac{1}{2}\Big)}{n!}$$

$$G_n(s) = \begin{cases} (-1)^{n/2}\cos\Big(s\dfrac{1+b}{2}\Big) & n=0,2,4,6,\cdots \\ (-1)^{(n+1)/2}\sin\Big(s\dfrac{1+b}{2}\Big) & n=1,3,5,7,\cdots \end{cases} \tag{33}$$

where $\Gamma(x)$ and $\mathrm{J}_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eqs.(31) and (32) into Eqs. (21)~(24), respectively, Eqs.(21) and (24) can be automatically satisfied, respectively. Then the remaining Eqs.(21) and (23) reduce, respectively, to

$$\sum_{n=0}^{\infty} a_n B_n \int_0^{\infty} \bar{\alpha}_0(s)\frac{\gamma}{s} G_n(s) \mathrm{J}_{n+1}\Big(s\frac{1-b}{2}\Big)\cdot$$

$$\cos(sx)\mathrm{d}s = \frac{\pi}{2\mu}\tau_0(1+\lambda) \tag{34}$$

$$\sum_{n=0}^{\infty} \Big(b_n - \frac{e_{15}}{\varepsilon_{11}}a_n\Big) B_n \int_0^{\infty} \bar{\alpha}_0(s) G_n(s)\cdot$$

$$\mathrm{J}_{n+1}\Big(s\frac{1-b}{2}\Big)\cos(sx)\mathrm{d}s = -\frac{\pi D_0}{2\varepsilon_{11}} \tag{35}$$

where $\lambda = \dfrac{e_{15}D_0}{\varepsilon_{11}\tau_0}$. For large s, the integrands of Eqs.(34) and (35) almost decrease exponentially. Hence, they can be evaluated numerically. Equations (34) and (35) can now be solved for the coefficients a_n and b_n by the Schmidt method[25] as can be seen in the Ref.[13].

5 NUMERICAL CALCULATIONS AND DISCUSSION

From Refs.[13, 28~30], it can be seen that the Schmidt method is satisfactory if the first ten terms in the infinite series in Eqs.(34) and (35) are retained. Although we can determine the entire perturbation stress field and the perturbation electric displacement field from coefficients a_n and b_n, it is important in fracture mechanics to determine the dynamic stress τ_{yz} and the electric displacement D_y in the vicinity of the crack tips. τ_{yz} and D_y along the crack line can be expressed, respectively, as

$$\tau_{yz}(x,0,t) = -\frac{2}{\pi}\sum_{n=0}^{\infty}\Big[\mu a_n B_n \int_0^{\infty} \bar{\alpha}_0(s)\frac{\gamma}{s}\cdot$$

$$\mathrm{J}_{n+1}\Big(s\frac{1-b}{2}\Big)\cos(xs)\mathrm{d}s + e_{15}\Big(b_n - \frac{e_{15}}{\varepsilon_{11}}a_n\Big)B_n\cdot$$

$$\int_0^{\infty} \bar{\alpha}_0(s)\mathrm{J}_{n+1}\Big(s\frac{1-b}{2}\Big)\cos(xs)\mathrm{d}s\Big] \tag{36}$$

$$D_y(x,0,t) = -\frac{2}{\pi}\sum_{n=0}^{\infty}(e_{15}a_n - \varepsilon_{11}b_n)B_n\cdot$$

$$\int_0^{\infty} \bar{\alpha}_0(s)\mathrm{J}_{n+1}\Big(s\frac{1-b}{2}\Big)\cos(xs)\mathrm{d}s \tag{37}$$

For $a = 0$ at $x = b$, 1, we have the classical stress and electric displacement singularities. However, so long as $a \neq 0$, the semi-infinite integration and the series in Eqs.(36) and (37) are convergent for any variable x. Eqs.(36) and (37) give a finite stress all along $y = 0$, so there is no stress and electric displacement singularities at the crack tips. At $b < x < 1$, τ_{yz}/τ_0 and D_y/D_0 are very close to unity, and for $x > 1$, τ_{yz}/τ_0 and D_y/D_0 take finite values diminishing from a finite value at $x = 1$ to zero at $x = \infty$. In all computations, the material constants are not considered, the parameters include the incident wave frequency, the wave velocity, the crack length and the lattice parameter in this paper. This is because the stress fields do not depend on the material constants. The results are plotted in Figs.2~9. The following observations can be made:

(1) The maximum perturbation stress and the perturbation electric displacement do not occur at the crack tip, but slightly away from it. This phenomenon has been thoroughly substantiated in Ref.[31]. The maximum stress and the maximum electric displacement are finite. The distance between the crack tip and the maximum stress point is very small, and it depends on the crack length and the lattice parameter. Unlike the classical piezoelectric theory solution, it is found that no stress and electric displacement singularities are present at the crack tip, and also the present results agree with the classical ones at places far away from the crack tip.

(2) The dynamic stress and electric displacement at the crack tip become infinite as the atomic distance $a \to 0$. This is the classical continuum limit of square root singularity. For the classical local theory, one can only obtain the stress and electric displacement intensity factors for the variation of ω/c.

(3) For the $a/\beta =$constant, viz., the atomic distance does not change, the values of the stress and electric displacements at the crack tip increase with increasing the crack length. From this fact, it can be shown that the piezoelectric materials with smaller cracks are more resistant to fracture than those with larger cracks.

(4) The significance of this result is that the fracture criteria are unified at both the macroscopic and microscopic scales, viz., it may solve the problem of any scale cracks.

(5) The left tip's stress and electric displacement are greater than the right tip's ones for the right crack. The stress and the electric displacement on the crack line become lower with increasing the distance between two cracks.

(6) The dimensionless perturbation stress is found to be independent of the electric loads and the material parameters. It just depends on the length of the crack, the lattice parameter, the circular frequency of the incident wave and the wave velocity. However, the perturbation electric field is found to be independent of the material parameters and the circular frequency of the incident wave and the wave velocity. It just depends on the length of the crack, the lattice parameter, which can be seen from Eqs.(34) and (35).

(7) The dynamic perturbation stress and the perturbation electric displacement at the crack tips tend to increase with the frequency, to reach a peak and then to decrease in magnitude. However, the dynamic perturbation stress and the perturbation electric displacement at the crack tips tend to decrease with increasing $a/2\beta$.

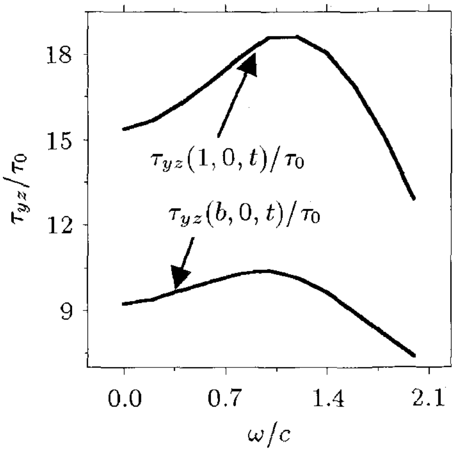

Fig.2 The variation with ω/c of the stress at the crack tips for $a/2\beta = 0.001$, $b = 0.1$, $\lambda = 0.2$

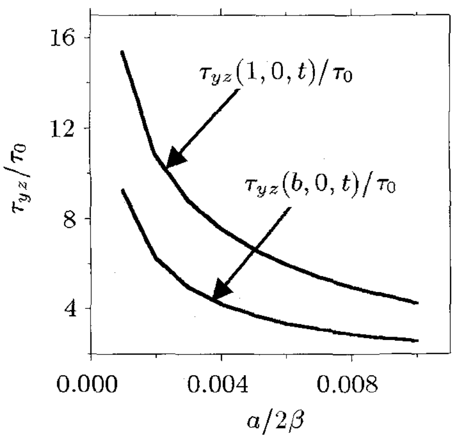

Fig.3 The variation with $a/2\beta$ of the stress at the crack tips for $\omega = 0$, $b = 0.1$, $\lambda = 0.2$

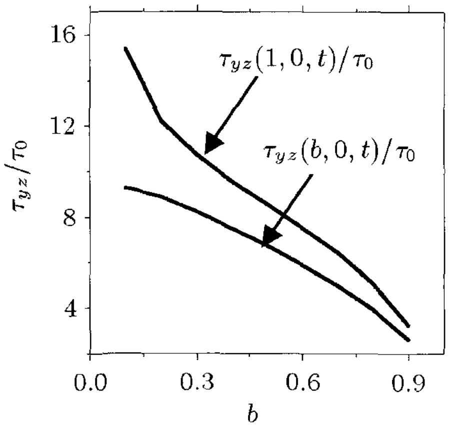

Fig.4 The variation with b of the stress at the crack tips for $\omega = 0$, $a/2\beta = 0.001$, $\lambda = 0.2$

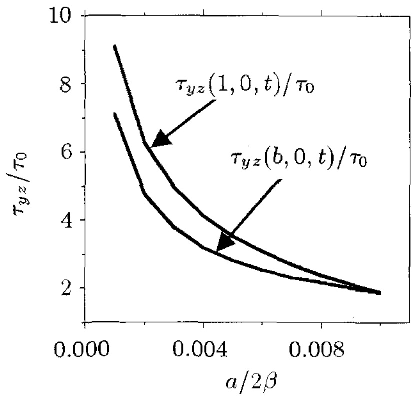

Fig.5 The variation with $a/2\beta$ of the stress at the crack tips for $\omega = 1.0$, $b = 0.5$, $\lambda = 0.2$

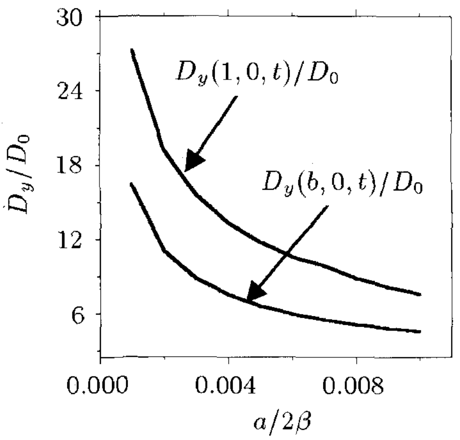

Fig.6 The electric displacement at the crack tips versus $a/2\beta$ for $\omega = 0$, $b = 0.1$, $\lambda = 0.2$

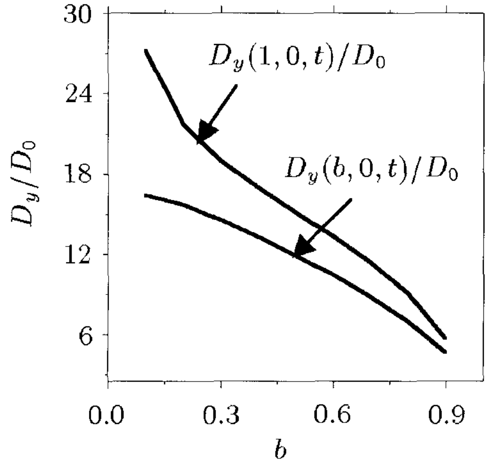

Fig.7 The electric displacement at the crack tips versus b for $\omega = 0$, $a/2\beta = 0.001$, $\lambda = 0.2$

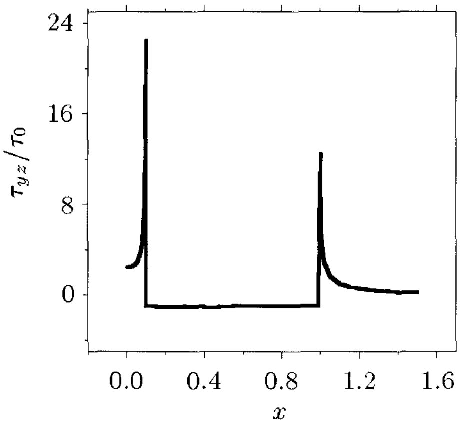

Fig.8 The variation of the stress on the crack line for $\omega = 1.0$, $b = 0.1$, $a/2\beta = 0.001$, $\lambda = 0.2$

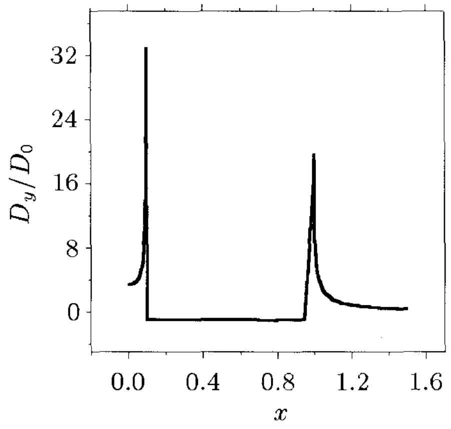

Fig.9 The variation of the electric displacement on crack line for $b = 0.1$, $a/2\beta = 0.001$, $\lambda = 0.2$

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